

Ph.D. THESIS PROPOSAL:

Reduced-Order Modeling and State Observer Design for Solid Oxide Electrolyzers

Tutors:

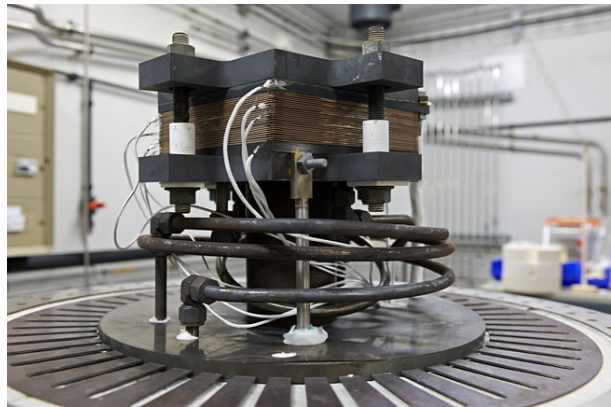
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Solid Oxide Cells (SOCs) are high-temperature electrochemical converters ($600\text{--}1000^{\circ}\text{C}$) capable of transforming chemical energy into electrical energy or, conversely, electrical energy into chemical energy. In fuel cell mode (SOFC), they generate electricity from hydrogen or synthetic gas, whereas in electrolysis mode (SOEC), they use electricity and heat to produce hydrogen or carbon monoxide from water and/or CO_2 . Such very high operating temperatures create a harsh environment for physical sensors. Since electrochemical reactions either consume or generate heat, dynamic models are required to understand and accurately describe the thermal states under which these devices operate [4].

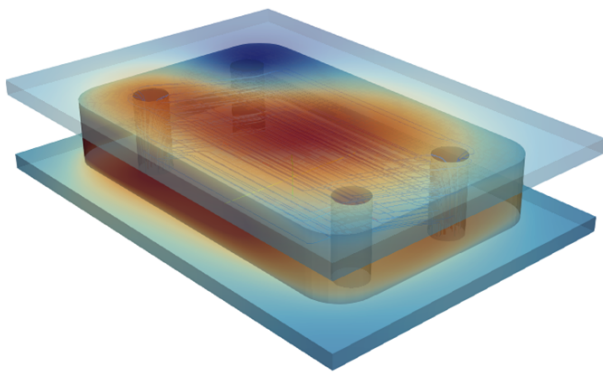


SOC Genvia [7]

As the technology reaches a higher level of maturity, the CEA supports GENVIA, a major industrial player in the sector, in the development of solid oxide electrolysis systems for water electrolysis applications. In this rapidly evolving context, which presents numerous scientific and technological challenges, the aforementioned models must be deployed for industrial applications, particularly for diagnostic purposes through virtual sensors based on state estimation techniques. To achieve short simulation times while preserving an accurate representation of the underlying physical phenomena, model reduction techniques and data assimilation methods leveraging the available measurements must be implemented. The estimation and observer design methodologies for large-scale systems developed at GIPSA-lab will provide a particularly relevant framework for addressing this challenge.

Numerical simulation of complex physical systems is a key tool for analysis, design, and optimization in modern engineering. High-fidelity models, typically based on partial differential equations and discretized using methods such as finite elements, can accurately capture multi-physics phenomena but often lead to very large-scale systems with prohibitive computational costs, especially in applications requiring repeated simulations, real-time decision-making, optimization, or state estimation. **Reduced-order modeling** aims to overcome this limitation by constructing low-dimensional representations that preserve the dominant physical dynamics while dramatically reducing computational complexity. Such models provide an effective com-

promise between accuracy and efficiency, enabling real-time simulation, control, virtual sensing, and digital twin applications for complex energy and industrial systems. Among the most widely used approaches, Proper Orthogonal Decomposition (POD) [5] and its snapshot-based variants provide efficient reduced representations, while Proper Generalized Decomposition (PGD) [10] offers a space-time decomposition but may require significant computational effort during model construction, limiting its industrial applicability. The project will therefore investigate innovative reduced-order modeling strategies combining **model reduction, machine learning, and optimization**, with the objective of developing computationally efficient models suitable for industrial applications, including simulation, control, and state estimation.



Heat model in SOC using POD [3]

Reduced-order models are particularly well suited to the design of **state observers** for complex systems, as the observer can compensate for modeling errors using measurements or reference trajectories, or alternatively exploit the model as a “virtual sensor” to estimate unmeasured variables, even in the presence of partially unknown inputs [8]. Such observers provide estimates of the system state. Early results combining observer design and POD can be found, for example, for the Navier-Stokes equations and for the control of systems governed by parabolic partial differential equations (PDE) [11]. Only a

few studies report real-time observer or estimator designs based on PGD [9]. The second objective of this project is to assess the capabilities and limitations of **observer design using reduced-order models** and to develop new dedicated state estimation methodologies. For example, multiple sensors placed in specific SOC locations motivate to consider the state-space SOC representation as a collectively observable linear time-invariant systems subject to unknown but bounded disturbance and measurement noise. Distributed interval estimation as proposed by [12] provides an interesting method along this line. This original scientific framework is expected to facilitate the integration of advanced model reduction techniques into SOC models, leading to significant improvements in both the accuracy and computational efficiency of complex-system simulations. An incremental validation strategy will be adopted, progressing from simple benchmark problems to industrial-scale case studies in order to progressively build expertise and ensure the robustness of the proposed approaches.

Infinite-dimensional observer design and adaptive estimation constitute a major challenge for the analysis and control of distributed physical systems governed by PDEs, particularly when unknown parameters and inputs are themselves spatially distributed and potentially time-varying. In contrast to finite-dimensional approaches obtained through spatial discretization, infinite-dimensional methods preserve the intrinsic structure of the system and provide strong theoretical guarantees regarding well-posedness, identifiability, and Lyapunov stability directly in functional spaces. This framework is especially relevant for physically demanding applications, such as heat transport in tokamak plasmas or distributed thermal dynamics in heat exchangers, where diffusion, advection, and multiphysics coupling phenomena cannot be reliably captured by low-dimensional models. Infinite-dimensional adaptive estimation [6] enables the **online identification** of diffusion coefficients and unknown source terms while ensuring convergence of the estimation error, thereby paving the way for robust diagnosis, control, and optimization strategies for complex distributed systems. Recent developments also make it possible to account for **nonlinear transport phenomena** and sparse mobile measurements [1]. A limitation on the previous method is that the observer cannot converge faster than information transport

in the direct model. This limitation is solved with a spectral boundary observer in [2], which uses PDE analysis to constrain the spectrum of the system's solution for parabolic equations. Such methods, stemming from infinite dimensional analysis, will be evaluated for SOC models and compared with the early lumping approaches mentioned in the previous paragraph. More precisely, new methods will be sought for **nonlinear parabolic equations** with the specific nonlinearities observed on SOC to guarantee asymptotic stability of the observers despite sparse measurements. Parameters adaptation will provide real time estimates of the transport properties. Further analysis will also include the coupling between heat and gas transport in the stack, thus necessitating to develop new observation methods for **mixed parabolic-hyperbolic systems** of equations.

Qualifications

- Bachelor and Master's degrees in Mechanical Engineering, Electrical Engineering, Applied Mathematics or a related field.
- Background knowledge in one or multiple fields: dynamic modeling, machine learning, advanced control, thermodynamics, fluid dynamics, simulation and implementation.
- Technical writing skills for scientific publications. Communication skills in English.
- A problem-solving-oriented mindset, self-motivation, initiative, resourcefulness, and dependability. Ability to both work independently and as part of a team.
- For international student, a minimum IELTS score of 7.0 or TOEFL iBT score of 92 is required.

How to apply

Interested applicants, please send your CV, copies of transcripts, recommendation letters, and previous publications (if any) to Prof. WITRANT and Dr. CARLIOZ using the subject line "PhD Position Application – SOC".

Other details

All qualified applicants are encouraged to apply. However, only candidates under consideration will be contacted. The starting date is as early as possible.

References

- [1] M. C. Belhadjoudja, M. Maghenem, and E. Witrant. State estimation for the kuramoto-sivashinsky equation using scanning outputs. *IEEE Transactions on Automatic Control*, 70(12):8398–8405, 2025.
- [2] M. C. Belhadjoudja, M. Maghenem, and E. Witrant. Spectral boundary observer for counter-flow heat exchangers. In *IEEE Conference on Decision and Control*, 2026.
- [3] T. Boucrelle. *Réduction de modèle pour représenter les phénomènes thermiques ayant lieu dans un électrolyseur haute température*. PhD thesis, Centrale Méditerranée, September 2025.
- [4] Q. Brillaut. *Modélisation thermique de stacks EHT : Approche et mise en œuvre d'outils de réduction de modèles numériques*. Theses, Université Grenoble Alpes, July 2025.
- [5] S. Chaturantabut and D. C. Sorensen. Nonlinear model reduction via discrete empirical interpolation. *SIAM Journal on Scientific Computing*, 32(5):2737–2764, 2010.

- [6] M. Ghousein and E. Witrant. Adaptive observer design for uncertain hyperbolic pdes coupled with uncertain ltv odes; application to refrigeration systems. *Automatica*, 154:111096, 2023.
- [7] Discover The Greentech. Genvia: industrialiser des électrolyseurs à oxyde solide à haute performance, 2026. <https://www.discoverthegreentech.com/encyclopedie/entreprises/genvia-industrialiser-des-electrolyseurs-a-oxyde-solide-a-haute-performance/> [Consulté le : 26 Août 2026].
- [8] S. Mechhoud, E. Witrant, L. Dugard, and D. Moreau. Estimation of heat source term and thermal diffusion in tokamak plasmas using a kalman filtering method in the early lumping approach. *IEEE Transactions on Control Systems Technology*, 23(2):449–463, 2015.
- [9] E. Nadal, F. Chinesta, P. Díez, F.J. Fuenmayor, and F.D. Denia. Real time parameter identification and solution reconstruction from experimental data using the proper generalized decomposition. *Computer Methods in Applied Mechanics and Engineering*, 296:113–128, 2015.
- [10] A. Nouy. A priori model reduction through proper generalized decomposition for solving time-dependent partial differential equations. *Computer Methods in Applied Mechanics and Engineering*, 199(23):1603–1626, 2010.
- [11] C. Xu, Y. Ou, and E. Schuster. Pod-based reduced order optimal control of parabolic pde systems via diffusivity-interior-boundary actuation. In *2007 46th IEEE Conference on Decision and Control*, pages 3519–3524, 2007.
- [12] J. Zhang, Z. Wang, N. Meslem, and T. Raïssi. Distributed interval estimation for continuous-time linear systems based on multi-hop decomposition and interval analysis. *Automatica*, 193:113199, 2026.